Non-Parametric Methods for Periodogram Analysis: Test Function Interrelations and Properties

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Abstract Numerical comparison of nine modifications of the non-parametric methods for periodogram analysis is carried out. The correlations among the values of the mean, variance, asymmetry, and excess of the test functions are determined.

1. Discussion

For "non-parametric" methods, the research of statistical properties of the test function was limited to evaluation of mathematical expectation for true and false frequency (e.g., Lafler and Kinman 1965). It is connected to the complexity, and frequently to the impossibility, of determination of analytical expression for the distribution function of the test function.

Analytic estimates of the mathematical expectation of the test function for different methods and of the dispersion of the test function by Lafler and Kinman (1965) were determined by Andronov and Chinarova (1997). The statistical distribution of the test functions computed for fixed data and various frequencies is significantly different from that computed for various data realizations. The histogram for the test functions is nearly symmetric for normally distributed uncorrelated data and is characterized by a distinctly negative asymmetry for noisy data with periodic components. Conditions for significant influence of the phase difference between the data onto the test functions are discussed.

A Detailed historical review and formulas for "non-parametric" methods were published by Terebizh (1992).

The test functions may be written as the sums of the terms V_i which define each particular modification. They include differences of phases $\Delta\phi_i = \phi_{i+1} - \phi_i$ and values of signal $\Delta x_i = x_{i+1} - x_i$. Here the observations must be sorted in nondecreasing phases. For calculation we used the following expressions:

1) $V_i = (\Delta x_i)^2$ (Lafler and Kinman, 1965)
2) $V_i = (\Delta x_i)^2 + (\Delta \phi_i)^2$ (Burke *et al.* 1970)
3) $V_i = (\Delta x_i / 2\mathrm{A})^2 + (\Delta \phi_i)^2$ (Dworetsky 1983)
4) $V_i = (\Delta x_i)^2 / ((\Delta \phi_i)^2 + \varepsilon_1^2)$ (Renson 1978)
5) $V_i = (\Delta x_i)^2 / (\Delta \phi_i + \varepsilon_1)$ (Renson 1978)
6) $V_i = (\Delta x_i)^2 / ((\Delta \phi_i)^2 + \varepsilon_2^2)$ (Renson 1978)
7) $V_i = (\Delta x_i)^2 / (\Delta \phi_i + \varepsilon_2)$ (Renson 1978)
8) $V_i = ((\Delta x_i / 2\mathrm{A})^2 + (\Delta \phi_i)^2)^{1/2}$ (Dworetsky 1983)
9) $V_i = |\Delta x_i|$ (Deeming 1970).

For the values of free parameters epsilon, introduced by Renson (1978), we have used two values, $\varepsilon_1 = 0.1$ and $\varepsilon_1 = 0.01$.

For illustration we have generated two sets of fifty data with random uniform distribution of the arguments. The "data" were generated (R) as the normally distributed noise and (RS) as the same noise with a sine component added with the same variance. The correlation coefficients between the test functions computed for 20,000 trial frequencies, the asymmetry *A*, excess *E*, ratio *W* of the variance of the test function to that expected for the χ_n^2 distribution are listed in Table 1. Also, the values of the ratio *U* of the depth of the most prominent minimum to the r.m.s. deviation of the test function are listed in Table 1.

The test-functions may be subdivided into two groups—similar to that by Lafler and Kinman (1965) and to that by Deeming (1970). The correlation coefficients for the test-functions within each group are close to unity for large number of data. For each method the parameter "U" is larger for the second group than for the first one and for the data with periodic signal "RS" than for the random data "R".

2. Addendum, 2006

These methods have particularly been applied to photographic (Odessa 7-camera astrograph) and CCD (Hipparcos-Tycho) observations.

References

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Table 1. Statistical characteristics (in %) of the periodograms 1–9 for the sets "R" (above the diagonal) and "RS" (below). For example, "1–3" corresponds to the correlation coefficient between the test functions "1" and "3", "U–1"—the value of U for the test function "1".

	1	2	3	4	5	6	7	8	9	*U*	*A*	*E*	*W*
1	—	100	100	99	98	61	77	90	89	347	–3	–13	47
2	100	—	100	99	98	61	77	90	89	347	–3	–13	47
3	100	100	—	99	98	62	77	90	89	346	–3	–13	44
4	99	99	99	—	100	66	81	89	88	342	–1	–12	48
5	99	99	99	100	—	73	87	89	87	337	0	–11	48
6	68	68	68	72	77	—	97	58	54	272	43	23	120
7	82	82	82	85	89	97	—	72	68	302	33	18	77
8	95	95	95	94	94	66	79	—	100	384	–14	–7	12
9	94	94	94	94	93	64	77	100	—	395	–15	–7	14
U	451	451	452	440	435	310	358	538	557				
A	–10	–10	–10	–10	–9	26	15	–18	–18				
E	2	2	2	1	2	7	7	9	10				
W	50	50	48	51	52	112	76	17	19				