Period Changes in RRc Stars

John R. Percy
Paul Jx Tan
Department of Astronomy and Astrophysics, University of Toronto, Toronto, ON M5S 3H4, Canada; john.percy@utoronto.ca

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Abstract We have used (O–C) analysis to study period changes in forty RRc stars in the GEOS (Groupe Européen d'Observation Stellaire) database of times of maximum of RR Lyrae stars. We find that many of the stars show approximately-linear period changes which are in agreement with the predictions of stellar evolution models. Other stars show period changes that are non-linear and/or larger than predicted by evolution models. Further long-term systematic, sustained observations may help to clarify the nature of the non-evolutionary changes.

1. Introduction

The period of a pulsating variable star may change for various reasons, including the evolution of the star; the pulsation period depends strongly on the radius of the star, and this may increase or decrease as a result of evolution. The change is very slow, but it is still observable because its effects on the observed times of maximum brightness are *cumulative*.

RR Lyrae stars have periods of about half a day, and visual amplitudes generally between 0.5 and 1.5 magnitudes. They are in the helium-burning phase of evolution, and lie on the horizontal branch in the Hertzsprung-Russell (HR) diagram. Their evolution is relatively rapid; they remain in this phase for only a few tens of millions of years. Their period changes have been studied for up to a century, especially those RR Lyrae stars in globular star clusters—for example, Jurcsik *et al.* (2012). For more information on RR Lyrae stars, see Smith (1995), or the mini-essays on RR Lyrae stars on the AAVSO's *Variable Star of the Season* webpage (http://www.aavso.org/vsots_archive).

RR Lyrae stars are subdivided into RRab stars which are pulsating in the fundamental mode, RRc stars which are pulsating in the first overtone, and RRd stars which are pulsating in both modes. RRc stars tend to lie on the blue (hot) side of the instability strip in the HR diagram. Some RR Lyrae stars show the *Blazhko effect* which is a slow, quasi-periodic change in the amplitude and shape of the light curve.

Percy *et al.* (2012) recently analyzed the period changes of fifty-nine RRab and RRc stars as a pilot research project and as an educational project, using times of maximum from the GEOS database. Le Borgne *et al.* (2007) had analyzed the period changes of some of the RRab stars a few years earlier. Walker (2010a, 2010b, 2010c, 2011) has carried out detailed analyses of the RRab stars RR Lyr, AR Her, BD Dra, and RW Dra (also EF Cnc, which is classified in the *General Catalogue of Variable Stars* (GCVS; Kholopov *et al.* 1985) as an eclipsing variable). Walker's four RRab stars have (O–C) diagrams with a complex mixture of period changes and Blazhko effect.

The period changes of RRc stars are more difficult to measure, because RRc stars have sinusoidal light curves with rounded maxima, whereas the RRab stars have light curves with sharp, easy-to-measure maxima. In the present paper, we analyze the period changes of forty RRc stars.

Since many of the times of maxima of RR Lyrae stars are measured by AAVSO observers, one important purpose of this paper is to provide observers with feedback on how their measurements are used in astronomical research. Another important purpose of this paper is to demonstrate how undergraduate students (such as co-author Tan) can carry out useful research with archival variable star data.

2. Data and analysis

We used times of maximum t(max) in a database of the Groupe Européen d'Observation Stellaire (GEOS), and the standard (O–C) method of analysis (see, for example, Percy 2007): we compare the observed time of maximum (O) with the calculated time (C), where C = t(0) + NP where t(0) is an initial epoch (time of maximum), P is the period (assumed to be constant), and N is an integer. A parabola is fit to the (O–C) diagram, and the rate of period change is proportional to the quadratic term in the best-fit parabola. The GEOS database is at: http://dbrr.ast.obs-mip.fr

The number of times of maximum in the database ranged from one to several hundred. For a satisfactory fit to the parabola, we require at least several dozen times of maximum, well-distributed in time with no lengthy gaps. That is because, if the gap is too long, it is often not possible to know the correct values of N corresponding to each time of maximum. We therefore examined all RRc stars in the database with at least one or two dozen times of maximum, and analyzed those without significant gaps. They are listed in Table 1. For completeness, the following stars were also examined, but not found suitable for analysis because of insufficient or poorly-distributed times of maximum: NU And, RW Ari, BS Boo, AK Com, AN Com, AR Com, AY Com, AZ Com, CE Com, CS Com, DQ Com, DR Com, DY Com, FI Com, FL Com, HY Com, V791 Cyg, V835 Cyg, V926 Cyg, V997 Cyg, EG Del, V410 Her, V458 Her, V806 Her, BB Leo, BR Leo, BX Leo, LO Lyr, LQ Lyr, LR Lyr, V518 Mon, V535 Mon, V558 Oph, V980 Oph, V2598 Oph, V2652 Oph, RU Psc, SX UMa, and AU Vir. The following stars in the table are suspected of being eclipsing binaries: UU Cam, BB CMi, V508 Cyg, KN Per, and perhaps V789 Cyg (see, for example, Hubscher *et al.* 2010). The sinusoidal light curve of an RRc star can be mistaken for that of a W UMa binary with equal minima.

The times of maximum have been determined by a wide variety of observers, using a wide variety of instruments and methods. Also, the database does not state how each time of maximum was measured from the light curve, but we presume that most were measured using Pogson's method, or by fitting a low-order polynomial to the light curve maximum. There is a significant benefit to using the *whole* light curve to determine the time of maximum, where this is possible.

Analysis was done with a least-squares routine, written in Python, based on the Levenberg-Marquardt algorithm (as implemented in the module scipy.optimize.leastsq); it gives the coefficients of the parabola, and their standard errors. *SciPy.org* is community-developed, open-source software for science, mathematics, and engineering.

3. Results

The results of the analysis are contained in Table 1, which lists: the name of the star, the number of times of maximum, the rate of period change in days per day, the characteristic evolution time τ, and the elements used for the analysis. We define τ as P/dP/dt; it is a measure of rate of evolution, but not necessarily the length of time that the star spends in the instability strip. The stars marked * have period changes which are significant at the 2σ level or better.

The stars in Table 1 are classified RRc in the GEOS database and in the GCVS. The stars with longer-than-average periods have amplitudes which are consistent with the RRc classification.

Some stars with fewer than twenty t(max) gave useful results, if the t(max) were well-distributed in time. The errors are large in some cases, but they still provide an upper limit to the rate of period change during the interval of observation.

Figures 1 through 4 show a selection of (O–C) diagrams. TV Boo (Figure 1) is well fit by a parabola, representing a period increase. AE Boo (Figure 2) can be fit by a parabola to yield an upper limit to the rate of period change, if dP/dt is constant. HY Com (Figure 3) is distinctly non-parabolic; it has a wavelike appearance. It can still be fit by a parabola to yield an upper limit to any underlying linear period change. SX UMa (Figure 4) is extremely complex, and is discussed below.

4. Notes on individual stars

The following comments apply to the (O–C) given in the GEOS database. TV Boo: an extra cycle was added to the t(max) at JD 2417703.3900. CQ Boo: the t(max) at JD 2435868.6300 was not used, because of the long gap following it. UY Cam: the t(max) at JD 2435565.2390 was not used because of the long gap following it. DY Com: unusually large scatter. RV CrB: very large, non-parabolic excursions in the (O–C) diagram. VZ Dra: there are four t(max) for which the cycle number had to be adjusted. LR Lyr: there are two t(max) for which the cycle number had to be adjusted, and the t(max) at JD 2436272.565 was not used because of the long gap preceding it. V535 Mon: the cycle number of the first t(max) was adjusted by one, and the last three t(max) were not used because of the long gap preceding them. ET Mus: one cycle was added to the t(max) at JD 2436732.3060. V2598 Oph: the data are sparse, but all t(max) are CCD measurements. SS Psc: the t(max) at JD 2392862.4750 was not used because of the large gap following it.

SX UMa is an unusual case; its (O–C) diagram (Figure 4) is extremely complex. The discontinuities occur because one or more cycles need to be (but have not been) added at those times. Even with such cycles added, however, the (O–C) diagram can only be represented by a number of different periods at different epochs. These periods are given in various publications and editions of the GCVS. The following are taken from the GEOS database: max = 2416200.486 + 0.3071148E (before JD2418800); max = 2418800.213 + 0.3071345E (JD2418800-22500); max = 2422653.5032 + 0.30711855E (JD2422500-26000); max = 2426400.418 + 0.3071555E (JD2426000-29500); max = 2430000.250 + 0.3071301E (JD2429500-34000); max = 2438508.751 + 0.3071363E (JD2434000-39000), where E is an integer.

Three other stars have distinctly non-parabolic (O–C) diagrams: RV CrB, RU Psc, and HY Com. These four stars all have relatively long periods: 0.307 day (SX UMa), 0.332 day (RV CrB), 0.390 day (RU Psc), and 0.449 day (HY Com).

5. Discussion and conclusions

The shorter-period stars (P less than 0.275 day) seem to have smaller rates of period change, and therefore longer evolution times; for four out of five, the period change is close to zero. These stars are of special interest, because there are very few stars in globular clusters with such short periods. The longer-period stars (P greater than 0.275 day) are equally divided into increasing and decreasing periods, though the longest-period stars generally tend to have decreasing periods. The rates range from ±1 to $\pm 1000 \times 10^{-10}$ day per day, or 0.04 to 40 days per million years—another commonly-used unit.

It is not simple to compare our observed period changes with those predicted by models of stellar evolution. Our stars are few in number; the number and range of the t(max) is limited; and their accuracy is not as great as for RRab stars. Furthermore: the predicted period changes depend on the mass and metallicity of the stars, and our stars (unlike those in a globular cluster) are field stars which have a range of metallicities and presumably of masses. The rates of period change, and the time that the star spends in the instability strip in the HR diagram, are especially sensitive to the mass of the star (Lee 1991). Lee predicts rates of ±0.2 d/Myr, including an allowance for scatter in the observed values, with a majority of the rates expected to be positive. Lee attributes the negative period changes to "random error," though it is not clear how such an error would arise, observationally.

About half of our values are significantly greater than the expected rates, and the same has been found in other studies. We note that some of our stars show distinctly non-parabolic (O–C) diagrams, and this is also not in agreement with evolutionary models. One suggested explanation is random mixing events which arise from composition instabilities in the cores of the stars (Sweigart and Renzini 1979). Observations of non-parabolic (O–C) diagrams may therefore shed light on the processes at work in these stars.

If the non-parabolic period changes are random, they may—over a long enough time—average out to the evolutionary rate of period change. One might then expect to see smaller average period changes for those RR Lyrae stars that have been observed for the longest time. In Table 1, the stars which have been observed for the shortest time tend to have large errors associated with the rate of period change. However, it is indeed true that the stars with the longest datasets (typically 30,000 days or more) have smaller rates of period change. Some pulsating variables show random cycle-to-cycle period fluctuations, but Percy *et al.* (2007) did not find such fluctuations in the one RR Lyrae star (XZ Dra) which they studied.

The strength of our data would improve if there were more measurements of t(max), if their time span were longer, and if they were more accurate (CCD measurements instead of visual, for instance). However, the data are sufficient to show that: (i) many of the stars show period changes whose nature and size are consistent with predictions of evolutionary models; (ii) some stars show period changes which do not agree, in nature and size, with predictions; (iii) systematic measurements of t(max) in these stars have both scientific and educational value.

The power of (O–C) analysis increases as the *square* of the length of the dataset, so further systematic, sustained observations of these stars would be scientifically valuable. To quote Lee (1991), referring to the comparison between observations and theory: "Observations of the RR Lyrae stars over the next century will undoubtedly help to clarify this problem further."

6. Acknowledgements

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Table 1. Rates of period change, and characteristic evolution times for RRc stars.

Star	*N*	*dP/dt (d/d)*	*τ(MY)*	*Elements*
NT And	53	2.24e–09 ± 1.55e–09	0.430 ± 0.298	2438642.4300 + 0.351661E
TV Boo*	97	1.82e–10 ± 1.50e–11	4.71 ± 0.389	2424609.5150 + 0.31255936E
AE Boo	57	7.87e–11 ± 5.22e–11	11.0 ± 7.26	2430388.2003 + 0.3148921E
BS Boo	20	–1.42e–08 ± 1.12e–08	0.0569 ± 0.0449	2437669.62 + 0.296149E
CQ Boo*	48	–2.07e–09 ± 7.11e–10	0.372 ± 0.128	2450948.5480 + 0.281883E
UY Cam*	54	1.02e–10 ± 1.87e–11	7.18 ± 1.32	2435565.2390 + 0.26704234E
ST CVn*	66	1.71e–10 ± 4.10e–11	5.26 ± 1.26	2440390.467 + 0.329051E
RZ Cep*	426	–1.32e–10 ± 1.61e–11	6.38 ± 0.777	2442635.374 + 0.3086853E
EZ Cep*	103	1.49e–10 ± 1.97e–11	6.95 ± 0.917	2426631.3700 + 0.378999E
U Com	36	–2.48e–11 ± 2.88e–11	32.4 ± 37.6	2424961.445 + 0.2927382E
AK Com	16	–3.85e–08 ± 4.5e–08	0.0258 ± 0.0301	2437737.63 + 0.36299E
AR Com	21	–2.01e–08 ± 4.07e–08	0.0397 ± 0.0806	2437764.45 + 0.291303E
AY Com	22	4.21e–08 ± 6.14e–08	0.0231 ± 0.0336	2437696.51 + 0.35455E
AZ Com	19	–5.04e–09 ± 6.57e–08	0.217 ± 2.83	2437696.59 + 0.39983E
CE Com*	19	5.73e–08 ± 2.14e–08	0.0145 ± 0.00542	2437668.44 + 0.304579E
CS Com*	22	–9.61e–08 ± 4.31e–08	0.00873 ± 0.00392	2437752.37 + 0.30645E
DQ Com	27	–5.83e–09 ± 3.33e–08	0.15 ± 0.859	2437764.24 + 0.320379E
DY Com*	19	8.73e–08 ± 3.99e–08	0.0112 ± 0.00511	2437785.28 + 0.35667E
FI Com	17	3.32e–08 ± 2.94e–08	0.0292 ± 0.0259	2437668.55 + 0.35344E
FL Com	26	2.25e–08 ± 2.7e–08	0.0442 ± 0.053	2437669.5 + 0.363775E
RV CrB*	282	8.61e–11 ± 3.92e–11	10.5 ± 14.5	2442926.3340 + 0.331565E
V791 Cyg*	54	–4.71e–09 ± 1.48e–09	0.197 ± 0.062	2433999.3804 + 0.33804949E

table continued on next page

Table 1. Rates of period change, and characteristic evolution times for RRc stars, cont.

Star	*N*	*dP/dt (d/d)*	*τ(MY)*	*Elements*
V926 Cyg	19	4.16e–11 ± 1.67e–10	20.2 ± 81.2	2433749.1992 + 0.30697965E
VZ Dra*	302	–4.57e–10 ± 7.38e–11	1.93 ± 0.311	2443534.7546 + 0.321025E
LS Her	95	1.49e–11 ± 2.23e–11	42.4 ± 63.4	2428004.9470 + 0.23080771E
BB Leo*	25	6.74e–08 ± 2.56e–08	0.0128 ± 0.00488	2437731.561 + 0.316028E
BR Leo*	18	–8.82e–08 ± 3.58e–08	0.0108 ± 0.00439	2437647.32 + 0.348384E
BX Leo*	17	–1.7e–09 ± 3.7e–10	0.584 ± 0.127	2438406.72 + 0.36286E
TV Lyn	74	2.87e–11 ± 1.76e–11	22.9 ± 14.1	2440950.9220 + 0.24065119E
LQ Lyr*	39	–7.97e–09 ± 3.58e–09	0.119 ± 0.0533	2429373.559 + 0.3451239E
LR Lyr	27	3.59–09 ± 4.93e–09	0.258 ± 0.355	2436272.565 + 0.338471E
V518 Mon	34	–1.03e–09 ± 5.78e–10	0.931 ± 0.526	2434769.522 + 0.34874473E
V535 Mon*	31	–5.48e–09 ± 5.43e–10	0.166 ± 0.0165	2434795.334 + 0.3328635E
ET Mus*	50	5.97e–09 ± 1.83e–09	0.105 ± 0.0322	2438536.2770 + 0.2296741E
V980 Oph*	16	–3.97e–10 ± 6.12e–11	2.39 ± 0.369	2435640.418 + 0.34696946E
V2598 Oph*	13	1.7e–09 ± 1.42e–10	0.623 ± 0.0518	2444374.383 + 0.38749054E
DH Peg	162	–1.16e–11 ± 3.46e–11	60.3 ± 180	2444463.5710 + 0.2555104E
SS Psc*	95	–7.76e–11 ± 1.90e–11	10.2 ± 2.49	2419130.3050 + 0.28779276E
T Sex*	52	4.02e–10 ± 9.79e–11	2.21 ± 0.538	2441384.3000 + 0.324698E
ER Sge*	55	1.29e–09 ± 1.60e–10	0.885 ± 0.11	2442585.9470 + 0.418431E
SX UMa	150	—	—	—

** Period change significant at the 2-sigma level or better.*

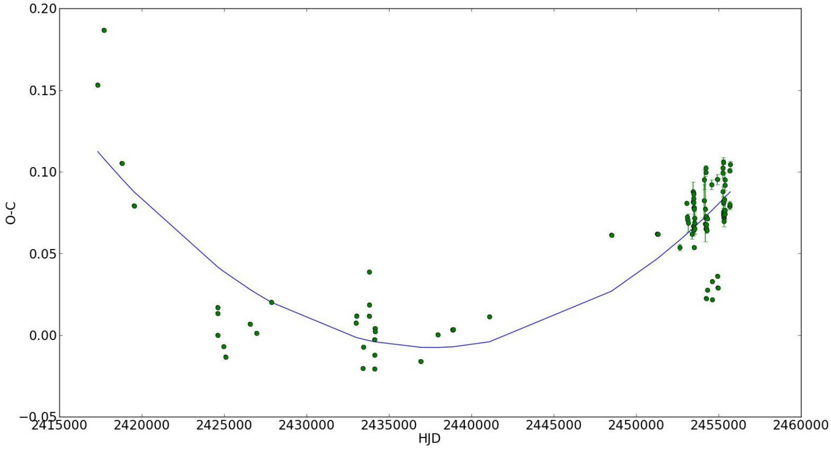

Figure 1. The (O–C) diagram of TV Boo. The diagram is parabolic, and corresponds to a period increase at a rate of $+1.82 \times 10^{-10}$ day/day.

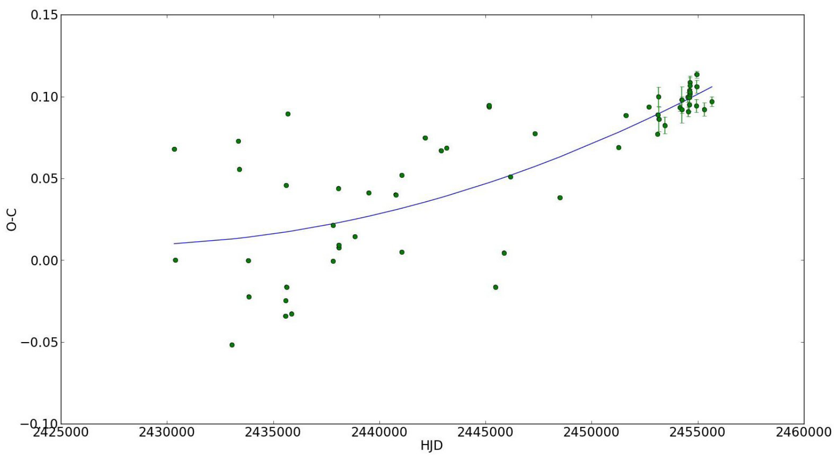

Figure 2. The (O–C) diagram of AE Boo. The diagram is rather scattered, but yields a period increase of $+7.87 \times 10^{-11}$ day/day with a σ which is comparable with the period change. It does, however, set a 3σ upper limit to the rate of period change.

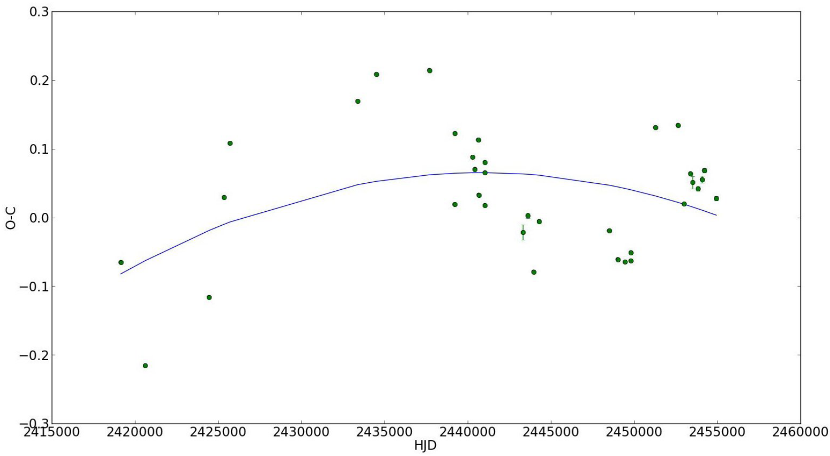

Figure 3. The (O–C) diagram of HY Com. The diagram does not appear to be sinusoidal, rather, it appears wavelike.

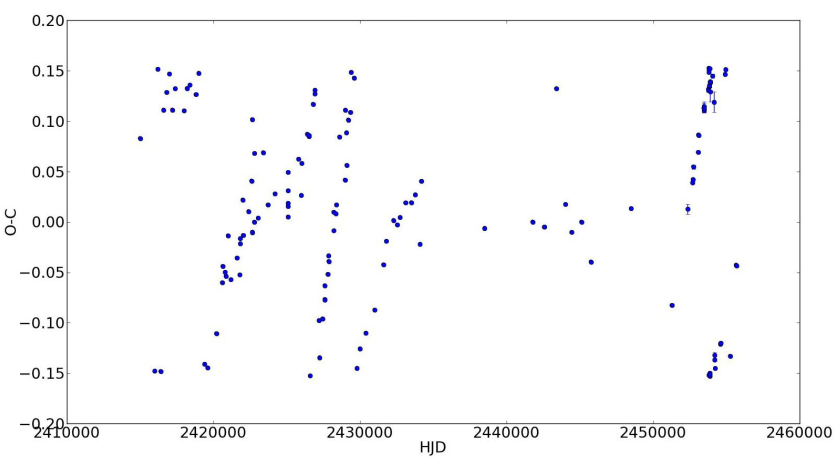

Figure 4. The (O–C) diagram of SX UMa. The diagram is very complex, and there are epochs at which extra cycles need to be added. See text for further discussion.