

Computing and using CCD Transformation Coefficients
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The purpose of transformation coefficients is to bring your photometry data more accurately in line with standard measurements. Each CCD camera and each set of filters responds slightly differently to different colors. The process of computing transformation coefficients provides a mathematical relationship between the response of your system and the response of “standard” photometric systems. While it is possible to measure star magnitudes without using transformation coefficients, you can improve the accuracy of your data by using them.

For those CCD photometrists who would like to determine their transformation coefficients and not be concerned about the underlying theory, this step by step guide might help. If you wish to learn more about transformation coefficients, please read Priscilla Benson’s paper: “CCD Transformation Coefficients,” available from the AAVSO website at <http://www.aavso.org/committees/ccdcoef.stm>.

Part I of this document describes the steps for determining your transformation coefficients.

Part II describes how to use the transformation coefficients when you perform photometry. A worked example is provided, which you should be sure to follow in detail.

Part I. Determining your transformation coefficients.

- Step 1. Choose a standard star field for observation.
- Step 2. Take images of the selected star field.
- Step 3. Apply bias, dark and flat frames to all the images.
- Step 4. Prepare an electronic or manual spreadsheet to record the data.
- Step 5. Compute several new columns in the spreadsheet.
- Step 6. Compute the relationships between some of these columns, and then compute the transformation coefficients.

Step 1. Choose a standard star field for observation. The star field should either be the M67 field described in Priscilla Benson's paper, or one of the many so-called Landolt fields. These are starfields distributed around the celestial equator (Declination = approx. 0), plus highly reliable photometric data for several stars in each field. You will need the finder charts as well as the photometric data. See **Appendix A** for information on how to find Landolt's finder charts and photometric data on the world wide web.

Step 2. Take images of the selected star field, using each of your filters – B, V, R, and I. You will need data from all the stars in all the colors, so be sure your fields are properly centered. Priscilla Benson recommends that you take several images on various nights, and that you recompute your transformation coefficients annually, to capture the evolving performance of your equipment.

Step 3. Apply bias, dark and flat frames to all the images with appropriate bias, dark and flat fields.

Step 4. Prepare an electronic or manual spreadsheet to record the data from the standard star field and from your measurements. A suggested layout for the spreadsheet can be found in the Benson paper, and is reproduced as Table 1 on the next page. Alternatively, software, such as IRAF, exists for working with this data. However, this paper will continue to use the spreadsheet approach.

Note that the column headings indicate the type of data in that column. For example, the columns labeled b, v, r and i contain instrumental magnitudes, hypothetically determined by you, the observer. Columns B-V, V-R, R-I and R are standard values given for the M67 stars.

You must compute columns labeled b-v, v-r and r-i. You do this by performing arithmetic on the data in columns b, v, r, and i. See Step 5 for more details.

Star Number	b	v	r	i	B-V	b-v	V-R	v-r	R-I	r-i
170	20.539	18.644	17.065	16.625		1.895	0.702	1.579	0.625	0.440
149	22.814	21.543	20.344	20.191		1.271	0.342	1.199	0.331	0.153
111	22.972	21.734	20.541	20.423		1.238	0.328	1.193	0.326	0.118
I-228	22.783	21.387	20.089	19.861		1.396	0.424	1.298	0.391	0.228
I-242	21.085	19.971	18.830	18.753		1.114	0.268	1.141	0.265	0.077
81	19.699	19.036	18.212	18.420	-0.098	0.663	-0.032	0.824	-0.036	-0.208
108	20.611	18.703	17.100	16.651	1.351	1.908	0.715	1.603	0.636	0.449
130	23.037	21.893	20.769	20.684	0.449	1.144	0.289	1.124	0.291	0.085
134	22.494	21.262	20.054	19.919	0.569	1.232	0.337	1.208	0.332	0.135
135	22.097	20.435	18.995	18.684	1.051	1.662	0.556	1.440	0.497	0.311
127	23.001	21.776	20.581	20.447	0.553	1.225	0.330	1.195	0.321	0.134
124	22.278	21.136	19.993	19.904	0.466	1.142	0.280	1.143	0.280	0.089
117	23.062	21.625	20.280	20.030	0.800	1.437	0.467	1.345	0.434	0.250

Table 1.

Throughout her paper Benson uses the following useful terminology:

- *Standard magnitudes* = B, V, R, I
- *Instrumental magnitudes* = b, v, r, i
- *Standard colors* = B-V, V-R, R-I. Note: The standard color (or "color index" V-I is becoming popular, but is not needed for determining transformation coefficients.)
- *Instrumental colors* = b-v, v-r, r-i
- *Standard differential magnitudes* =
 - ΔB , the difference between the standard B magnitude of any star and the standard B magnitude of any other star
 - ΔV , the difference between the standard V magnitude of any star and the standard V magnitude of any other star
 - ΔR , the difference between the standard R magnitude of any star and the standard R magnitude of any other star
 - ΔI , the difference between the standard I magnitude of any star and the standard I magnitude of any other star
- *Instrumental differential magnitudes*
 - Δb , the difference between the instrumental b magnitude of any star and the instrumental b magnitude of any other star
 - Δv , the difference between the instrumental v magnitude of any star and the instrumental v magnitude of any other star
 - Δr , the difference between the instrumental r magnitude of any star and the instrumental r magnitude of any other star
 - Δi , the difference between the instrumental i magnitude of any star and the instrumental i magnitude of any other star
- *Standard differential colors* =
 - $\Delta(B-V)$: B-V for any star, subtracted from B-V for any other star
 - $\Delta(V-R)$: V-R for any star, subtracted from V-R for any other star
 - $\Delta(R-I)$: R-I for any star, subtracted from R-I for any other star
- *Instrumental differential colors* =
 - $\Delta(b-v)$: b-v for any star, subtracted from b-v for any other star
 - $\Delta(v-r)$: v-r for any star, subtracted from v-r for any other star
 - $\Delta(r-i)$: r-i for any star, subtracted from r-i for any other star

There are two more terms (for which Benson does not provide terminology):

- V-v, instrumental v magnitude of a star subtracted from standard V magnitude for the same star.
- R-r, instrumental r magnitude of a star subtracted from standard R magnitude for the same star.

Both of these terms are used for transforming instrumental measurements to standard magnitudes, as is explained in **Part II**.

Step 5. Compute several new columns based on the standard and instrumental data:

Referring to Table 1, the instrumental colors in the column labeled “b-v” are computed by subtracting cells in the column labeled “v” from corresponding cells in the column labeled “b”. As a check, note that for Star 170, b-v is 1.895, determined by computing $20.539 - 18.644$, the values of b and v for Star 170.

The column of instrumental colors “v-r” is computed by subtracting cells in column “r” from corresponding cells in column “v”.

The column of instrumental colors “r-i” is computed by subtracting cells in column “i” from corresponding cells in column “r”. For the rest of this paper, we will not bother to enclose column headings in quotation marks.

Column R-r is computed by subtracting cells in column r from corresponding cells in column R.

The column for standard magnitudes V is computed by adding cells in column V-R to corresponding cells in column R.

Column V-v is computed by subtracting cells in column v from corresponding cells in the newly computed column V.

Step 6. By plotting the points in certain columns against corresponding points in other columns, you should see approximate straight lines. Graph 1 is an example. Benson’s paper includes several other examples.

The numerical values of the slopes of these lines are the numbers needed to determine the transformation coefficients. In practice, some of these plots do not fit straight lines very well, even in Benson’s example. Still, we will compute the best fit straight lines and use the slopes of those lines to compute the transformation coefficients.

Benson suggests using software to compute the slopes of the best fit lines. If your software lacks an easy computation, you can use the formula she provides near the end of paragraph 6 of her document. However, if you are using an electronic spreadsheet program such as Excel, you already have the necessary formula built-in. For Excel, the built-in formula is

$$=SLOPE(y\text{-values}, x\text{-values})$$

An easy mistake in this process is to use the wrong convention regarding which values are x-values, and which are y-values. Normally the tendency will be to use the x-axis for the independent variable or the more directly known value. However, Table 2 shows the x-y conventions that must be used to be consistent with Benson's paper (see also the first paragraph of section 6 Benson's paper). Table 2 also shows how to compute the transformation coefficients, once the slopes have been determined.

x-values	y-values	Compute and call the slope:	Compute the transformation coefficient
B-V	b-v	m_{bv}	$T_{bv} = 1/ m_{bv}$
V-R	v-r	m_{vr}	$T_{vr} = 1/ m_{vr}$
R-I	r-i	m_{ri}	$T_{ri} = 1/ m_{ri}$
V-R	V-v	m_v	$T_v = m_v$
R-I	R-r	m_r	$T_r = m_r$

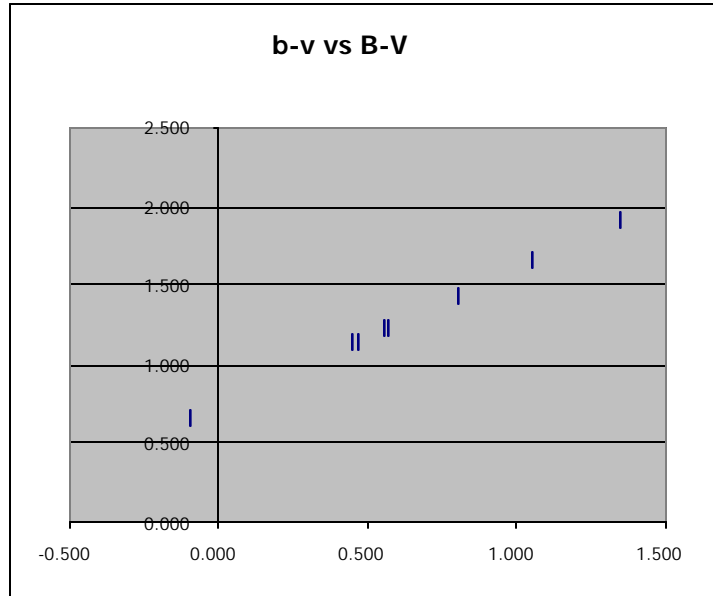
Table 2.

Let's follow the computations for T_{bv} . According to Table 2, we will need the data in columns B-V and b-v. This data is reproduced here in Table 3.

Star Number	B-V	b-v
170		1.895
149		1.271
111		1.238
I-228		1.396
I-242		1.114
81	-0.098	0.663
108	1.351	1.908
130	0.449	1.144
134	0.569	1.232
135	1.051	1.662
127	0.553	1.225
124	0.466	1.142
117	0.800	1.437

Table 3.

Benson's paper does not provide B-V data for stars 170 through I-242, so we will restrict ourselves to stars 81 through 117. A plot of the B-V values (x-axis) against the b-v values (y axis) is shown in Graph 1.



Graph 1.

The slope of the best fit straight line to these points can be computed using the Excel function

$$=SLOPE(y-values, x-values)$$

where *y-values* refers to the data in the b-v column and *x-values* refers to the data in the column labeled B-V. In this case the value of the slope is 0.8623.

Since the equation for T_{bv} is $T_{bv} = 1/m_{bv}$, we have

$$T_{bv} = \frac{1}{m_{bv}} = \frac{1}{0.8623} = 1.1597$$

Table 4 shows all the worked out values from the Benson's sample calculations of transformation coefficients for M67, starting with the hypothetical instrumental magnitudes shown in Table 1.

x-values	y-values	Slopes:	Computed transformation coefficients
B-V	b-v	0.8623	$T_{bv} = 1/m_{bv} = 1.1597$
V-R	v-r	1.0473	$T_{vr} = 1/m_{vr} = 0.950$
R-I	r-i	0.9948	$T_{ri} = 1/m_{ri} = 1.0053$
V-R	V-v	0.0543	$T_v = m_v = 0.0543$
R-I	R-r	0.1113	$T_r = m_r = 0.1113$

Table 4

Once you have completed Steps 1-6 you will have filled in your version of Table 4 with your own transformation coefficients. You should report these to the AAVSO CCD

committee by either mailing in Benson's form (the 7th page of Benson's paper) or by contacting Gary Walker bailyhill@aol.com.

Note that formulas Benson provides on the form are intended to be used when actually performing photometry and are therefore not relevant to the form. Also, some of these equations contain typographical errors. The correct forms of these equations are shown below in Part II.

Part II. Using your transformation coefficients for photometry

The purpose of transformation coefficients is to transform your instrumental differential colors to standard differential colors. From there you can compute standard magnitudes. The steps involved are:

- Step 1. Take images of the variable star.
- Step 2. Extract instrumental magnitudes from your images.
- Step 3. Compute differential instrumental magnitudes.
- Step 4. Compute differential instrumental colors.
- Step 5. Transform the differential instrumental colors to standard differential colors.
- Step 6. Compute standard differential magnitudes.
- Step 7. Compute standard magnitudes.

The process is represented graphically in Diagram 1:

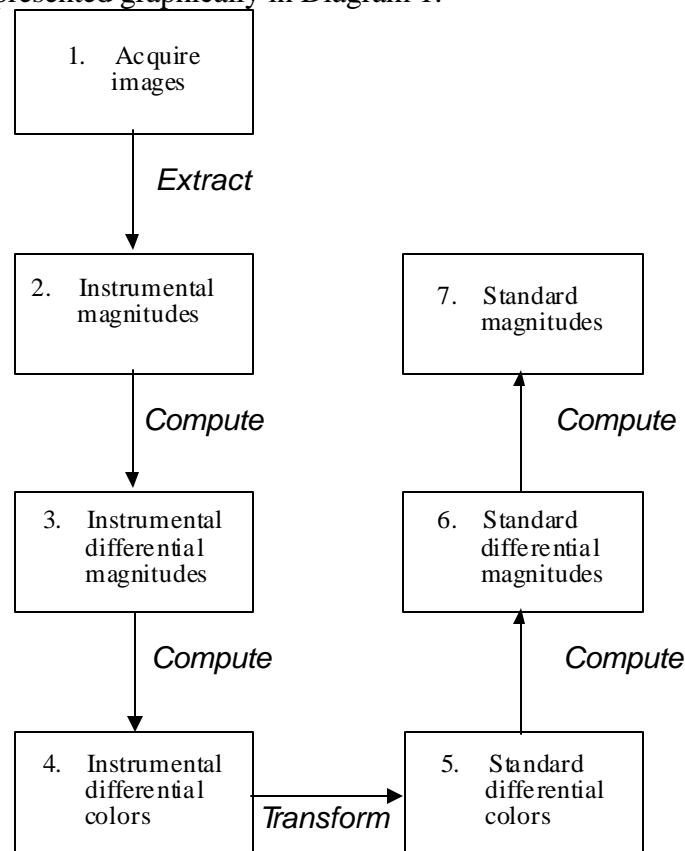


Diagram 1. Differential photometry process.

To illustrate this and the other steps in Part II, we will use a slightly modified version of the Benson example called “Sample Calculation of Transformed Magnitudes” in her paper. Her example assumes you have imaged RR Boo and are going to use the Transformation coefficients to develop the standard magnitudes (B, V, R, I) for the star.

Benson's example omits data in B and b. I have added standard values of B from the AAVSO CCD finder chart for RR Boo, and I have also added hypothetical instrumental values for b.

Step 1. Take images of the variable star.

You start by imaging RR Boo, including in your field the 3 comp stars: comp 2, comp 4 and comp 6. Imaging should be performed in four colors: B, V, R, and I. Apply bias, dark and flat frames to the images.

Step 2. Extract and record instrumental magnitudes.

Use your image processing software to extract magnitudes of the variable star and a few comp stars from each of the b, v, r and i images. Arrange your data in a spreadsheet form, as in Table 5.

[For more information on how to extract instrumental magnitudes, see either the notes to Arne Henden's tutorial on Precision CCD Photometry (from the 90th AAVSO Annual Meeting in Somerville, Mass) or see Chapter 8 of "Handbook of Astronomical Image Processing" by Richard Berry and James Burnell, Willmann-Bell, Inc. 2000. Of special interest are sections 8.2.1 – 8.2.4. Also see **Appendix B.**]

Star	b	v	r	i
RR Boo	22.772	21.284	18.508	16.974
comp2	21.488	19.118	17.599	17.519
comp4	22.987	22.516	20.999	20.969
comp6	23.502	23.151	21.625	21.649

Table 5. Instrumental magnitudes

Step 3. Compute and record instrumental differential magnitudes.

Recall that an instrumental differential magnitude is the difference between the instrumental magnitude of a star (as measured in one particular color) and the instrumental magnitude of any other star, as measured in the same color.

	Δb	Δv	Δr	Δi
RR-boo - comp2	1.284	2.166	0.909	-0.545
RR Boo - comp4	-0.215	-1.232	-2.491	-3.995
RR Boo - comp6	-0.730	-1.867	-3.117	-4.675

Table 6. Instrumental differential magnitudes.

To check your math, note that the value 2.166 comes from subtracting the instrumental v magnitude of comp 2 (see Table 5) from the instrumental v magnitude of RR Boo, or $21.284 - 19.118 = 2.166$. Note that in Benson's paper these columns are incorrectly notated: v, r and i.

Step 4. Compute the instrumental differential colors.

Recall that an instrumental differential color is the difference between the instrumental color of one star and the same instrumental color of another star. For example, the instrumental differential color $\Delta(b-v)$ of RR Boo – comp2 is the value of b-v for RR Boo minus the value of b-v for comp2.

	$\Delta(b-v)$	$\Delta(v-r)$	$\Delta(r-i)$
RR Boo - comp2	-0.882	1.257	1.454
RR Boo - comp4	1.017	1.259	1.504
RR Boo - comp6	1.137	1.250	1.558

Table 7. Instrumental differential colors

To check your math, note that the value 1.257 in Table 7 comes from subtracting 0.909 (RR-Boo - comp2's r instrumental differential magnitude in Table 6) from 2.166 (RR-Boo - comp2's v instrumental differential magnitude in Table 6.) Note that in Benson's paper these columns are incorrectly notated v-r and r-i.

In Benson's example (together with our addition to the example) she has assumed that our transformation coefficients are:

$$\begin{aligned} T_{bv} &= 1.16 \\ T_{vr} &= 0.95 \\ T_{ri} &= 1.00 \\ T_r &= 0.09 \end{aligned}$$

We use the transformation coefficients (T_{bv} , T_{vr} and T_{ri}) to transform our instrumental differential colors to standard instrumental differential colors. This is done by:

- multiplying $\Delta(b-v)$ by T_{bv} (giving us $\Delta(B-V)$)
- multiplying $\Delta(v-r)$ by T_{vr} (giving us $\Delta(V-R)$)
- multiplying the $\Delta(r-i)$ values by T_{ri} (giving us $\Delta(R-I)$).

Benson's formulas are given as:

$$\begin{aligned} V-R &= T_{vr} \cdot (v-r) \\ R-I &= T_{ri} \cdot (r-i) \\ R &= r + T_r \cdot (R-I) \end{aligned}$$

The formulas should read:

$$\begin{aligned} \Delta(B-V) &= T_{bv} \cdot \Delta(b-v) \\ \Delta(V-R) &= T_{vr} \cdot \Delta(v-r) \\ \Delta(R-I) &= T_{ri} \cdot \Delta(r-i) \\ \Delta R &= \Delta r + T_r \cdot \Delta(r-i) \end{aligned}$$

In the case of Benson's example, our transformed differential colors are:

	$\Delta(V-R)$	$\Delta(R-I)$	ΔR
RR Boo - Comp2	1.194	1.454	1.040
RR Boo - Comp4	1.196	1.504	-2.356
RR Boo - Comp6	1.188	1.558	-2.977

To check your math, note that 1.194 comes from multiplying RR Boo – Comp2's v-r value of 1.257 by the transformation coefficient $T_{vr} = 0.95$. Note that in Benson's paper these columns are labeled: V-R, R=I and R.

Now that we have our transformed differential colors we can compute our transformed differential magnitudes for RR Boo, based on Comp2, Comp4 and Comp6. We use these formulas (not given in Benson's paper):

$$\Delta V = \Delta(V-R) + \Delta R$$

$$\Delta R = \Delta R \text{ (already computed)}$$

$$\Delta I = \Delta R - \Delta(R-I)$$

The missing formula for ΔB is:

$$\Delta B = \Delta(B-V) + \Delta V$$

The Benson example values are:

	ΔV	ΔR	ΔI
RR Boo - Comp2	2.234	1.040	-0.414
RR Boo - Comp4	-1.160	-2.356	-3.860
RR Boo - Comp6	-1.789	-2.977	-4.535

To check your math, note that $\Delta V = 2.234$ comes from adding the transformed differential color $\Delta(V-R)=1.194$ of RR Boo – Comp2 to the transformed differential magnitude $\Delta R=1.040$ of RR Boo – Comp2. Also, $\Delta I = -0.414$ comes from subtracting $\Delta(R-I)=1.454$ from $\Delta R=1.040$.

Our final step is to compute the color magnitudes of RR Boo, based on each of the 3 comparison stars. To do this we recognize that:

$$\Delta V = (V \text{ magnitude of RR Boo}) - (V \text{ magnitude of Comp star})$$

Since we have been given (V magnitude of Comp star), for each Comp star, we can add that value to ΔV :

$$\Delta V + (V \text{ magnitude of Comp star}) =$$

$(V \text{ magnitude of RR Boo}) - (V \text{ magnitude of Comp star}) + (V \text{ magnitude of Comp star})$, we have:

$$\Delta V + (V \text{ magnitude of Comp star}) = (V \text{ magnitude of RR Boo}) = V$$

Likewise for the other colors:

$$\Delta R + (\text{R magnitude of Comp star}) = (\text{R magnitude of RR Boo}) = R$$

$$\Delta B + (\text{B magnitude of Comp star}) = (\text{B magnitude of RR Boo}) = B$$

$$\Delta I + (\text{I magnitude of Comp star}) = (\text{I magnitude of RR Boo}) = I$$

The Benson example values of magnitudes of RR Boo are:

	V	R	I
Based on Comp2	11.895	10.086	8.052
Based on Comp4	11.913	10.166	8.120
Based on Comp6	11.967	10.177	8.090
Average	11.925	10.143	8.087
Std dev of values	0.030	0.041	0.028

To check your math, note that V, Based on Comp2, equals 11.895. This comes from adding $\Delta V=2.234$ for Comp2 to the standard V value for Comp 2, which is 9.661.

To increase the precision of our estimate of the color magnitudes for RR, we could average the values for V based on each of the 3 Comp stars. In the example, the average for V is 11.925. I have added the standard deviation of these averages as an estimate of the error. There has recently been some discussion amongst AAVSO members about the advantages and disadvantages of using multiple comp stars and averaging the result, versus using a single comp star. For more discussion of this topic, consult the AAVSO CCD committee.

Appendix A. Locating Landolt Standard Fields.

The key document identifying these fields is a paper written by Arlo U. Landolt, and published in The Astronomical Journal, Volume 104, Number 1, July 1992. This paper is 88 pages long and describes his methods and results in determining photometric data for 526 stars centered on the celestial equator.

The paper includes finder charts and a table of photometric data for all of these stars. Unfortunately the paper is not available in an electronically editable form. It can be downloaded from the internet as a .pdf file, but no text, graphical data, or numerical data can be extracted from it easily.

For the complete pdf file, go to

<http://astron.berkeley.edu/~kalas/disksite/pages/resources.html>

then click on pdf. . The entire 58MB pdf file will download to your hard drive. From this same site you can also download the paper in postscript format.

Because of the importance of the paper, the finder charts and photometric data have been made available in machine readable form at several websites. The data is intended for professional use on large telescopes, and may be difficult for amateur use. Here are a few:

<http://www.ls.eso.org/lasilla/Telescopes/2p2T/Landolt/>

This site is managed by La Silla Observatory in Chile. At this site you will see a long list of stars arrange by RA. Landolt's standard photometric data is provided for each star. Also for each star a link is provided to a finder chart that includes that star and other nearby stars.

Another similar website is managed by Lick Observatory:

<http://www.ucolick.org/~jharris/landolt/>.

Landolt's scanned paper can be found at http://adsbit.harvard.edu/cgi-bin/nph-article_query?1992AJ%2E%2E%2E%2E104%2E%2E340L. However, you will have to read the paper online at this site.

Appendix B. About instrumental magnitudes.

Many AAVSO members have read "Handbook of Astronomical Image Processing" by Richard Berry and James Burnell". Chapter 8: Photometry, covers many of the topics of interest to AAVSO members. In my copy (First English Edition, Second Printing, 2001) there appears to be a typographical error in an important formula in Section 8.2.4, Equation 8.16. The formula for raw instrumental magnitude is given as:

$$Z = m + 2.5 \bullet \log \left(\frac{(\bar{P}_{star} - \bar{P}_{sky}) N_{star}}{t} \right) + Z$$

The correct equation should be:

$$m = -2.5 \bullet \log \left(\frac{(\bar{P}_{star} - \bar{P}_{sky}) N_{star}}{t} \right) + Z$$

We can use this equation to compute instrumental magnitude as mentioned in this document. However, some work find a realistic value for the arbitrary zero point Z will be worthwhile to bring the instrumental magnitude into a range comparable to the given comp star magnitudes.

An alternative method for computing instrumental magnitude avoids this difficulty. Using Berry's equation 8.1 in section 8.1.1:

$$m_1 - m_2 = -2.5 \log \left(\frac{F_1}{F_2} \right),$$

In this equation m_1 and m_2 are magnitudes of stars in a given field, and F_1 and F_2 are measures of the fluxes of these stars. Determining the flux requires some care, and discussed by Berry in sections 8.2 - 8.2.3. Also, see very helpful material in Arne Henden's tutorial on Precision CCD Photometry (from the 90th AAVSO Annual Meeting in Somerville, Mass.)

Select a star of known magnitude and assign that magnitude to m_2 . Then the instrumental magnitude m_1 for a star will be:

$$m_1 = -2.5 \log \left(\frac{F_1}{F_2} \right) + m_2$$